TRANSPORT IN A TURBULENT VORTEX RING

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Transport in a turbulent vortex ring is considered; the mass of a passive impurity transported by the vortex is found as a function of distance traveled via a formula that agrees closely with experiment. A method is indicated for filling the part of the vortex that transports the impurity without loss. The effects of the following factors on the transport have been examined: initial Reynolds number, roughness in walls of the exit hole, and density difference between the solution and the medium. The transport of aerosols and suspended particles by vortex rings is considered. Two methods of transport measurement are compared.

1. Previous papers [1, 2] have dealt with material transport by vortex rings; [1] deals with the turbulent diffusion of a vortex, and it was concluded that a certain region in a vortex can transport material without loss. Results have also been given [2] from experiments on transport by vortex rings as examined by photometry. Laminar and turbulent vortices with initial Reynolds numbers Re_0 not more than $5 \cdot 10^3$ were examined (these values were determined from the initial velocity u_0 and the radius R_0).

Visual observations on colored turbulent vortex rings showed that they leave behind a characteristic trace of the coloring matter, and the dye in the vortex atmosphere [3] rapidly passes into the trace. The dye persists virtually to the end of the motion in the pronounced torroidal region. It is assumed that this region consists of part of the boundary layer formed on expulsion of the liquid from the generator. Cinephotography has been applied to vortices with only the inner surface of the source tube colored, and the results confirm the above (see Fig. 1). This experiment also showed that the core of the vortex roughly coincides with the region that carries the passive impurity without loss in the initial stage.

The following model can be considered for transport by turbulent vortex ring; we assume that: 1) the impurity concentration c in the atmosphere is kept uniform on account of the turbulent mixing (although this concentration is somewhat higher in the region adjoining the core than in the rest of the atmosphere); 2) the impurity concentration in the trace (mean radius denoted by δ, Fig. 2), which enters from the atmosphere on account of turbulent diffusion, is also uniform, but is lower than in the atmosphere on account of more pronounced mixing with the surrounding medium (the value is nc, where $n < 1$ is a constant coefficient; 3) no impurity is lost from the torroidal region that coincides with the core in the initial stage.

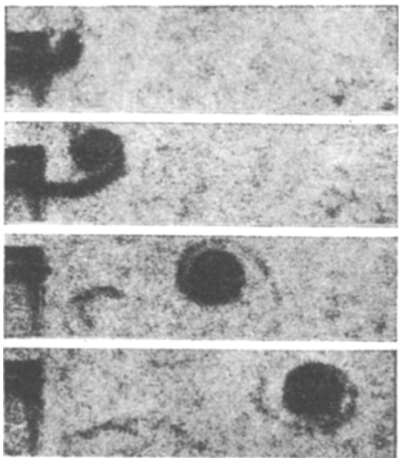

Fig. 1

We denote by M_0' and M' the initial and current masses of the impurity in the atmosphere; the above scheme gives us the mass leaving the atmosphere or the trace in terms of

$$dM' / dt = -ncu\pi\delta^2 \qquad (1.1)$$

where u is the vortex speed. As the radius R of the vortex increases linearly on account of the turbulent viscosity [4]:

$$R = R_0 + \alpha L \qquad (1.2)$$

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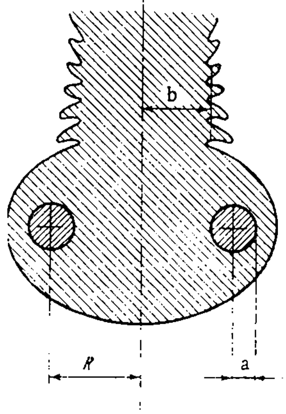

Fig. 2

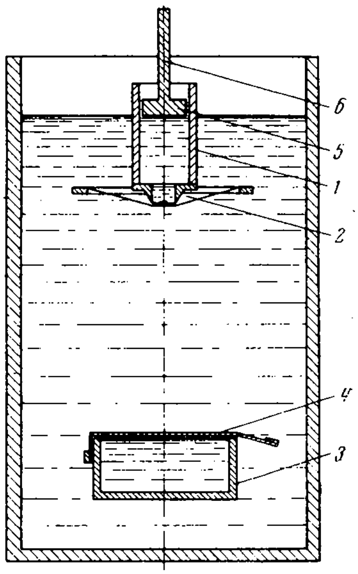

Fig. 3

where $\alpha \sim 10^{-2} - 10^{-3}$ is a constant during the motion that is dependent on the initial conditions, while L is the distance traveled by the vortex. We neglect the initial volume of the core by comparison with the total volume V, which gives $M' = cAR^3$, where $A = V/R^3$ is a constant coefficient. As $dL/dt = u$, $\delta^2 = BR^2/\pi$ (B is a constant), we get from Eq. (1.1) that

$$dM' / dR = -nBM' / \alpha AR \tag{1.3}$$

Then

$$M' = M_0' (R_0 / R)^{3\beta}, \quad \beta = nB / A \tag{1.4}$$

The mass M transported by the vortex is

$$M = m_0 + M_0' (1 + \alpha L / R_0)^{-3\beta} \tag{1.5}$$

where m_0 is the initial mass in the core.

2. To test Eq. (1.5) and to determine m_0 and β we made experiments with turbulent vortex rings with initial Reynolds numbers varying from $4 \cdot 10^3$ to $4 \cdot 10^4$; Fig. 3 shows the apparatus.

The experiment was carried out as follows. A measured amount of colored liquid was placed in the vortex generator 1, whose exit hole was closed by the stop 2; the dye was fluosang, which allowed us to make measurements with an FÉK-56 photoelectric colorimeter with appropriate sizes for the vortex and the trap 3. After the rubber diaphragm 2 had ruptured, which provided for uniformity in the experiments, the vortex was initiated; when the vortex had entered the trap 3 at a set distance from the source, the rubber shutter 4 operated, which sealed it. The transmission coefficient of the solution in the trap was measured with the FÉK-56. The concentration was deduced from the calibration curve. The amount of dye was determined from the known volume of the trap.

The generator produced volumes of liquid such as to be completely incorporated in the vortices; the vortex formed within distances of 4-5 R_0 from the end of the nozzle. Tests showed that the colored liquid formed more than 95% of the total volume of the vortex. The constant stroke of the piston 5 in the source provided a constant volume in each case. Eddies were selected and monitored for constant initial speed by recording the piston motion on an oscilloscope with a variable resistor coupled to the rod 6. The piston speed was varied, but was constant in each series. The diameter d of the exit hole and the piston stroke were adjusted so that the length l of the ejected jet did not exceed 4d. This was the condition for the jet to enter the vortex completely. The initial vortex radius R_0 and the constant α were derived by cinephotography, using a Konvas camera with a synchronous motor operating at 25 frames per second.

3. Curve 1 of Fig. 4 shows a typical relationship between the transported mass and the distance trav-

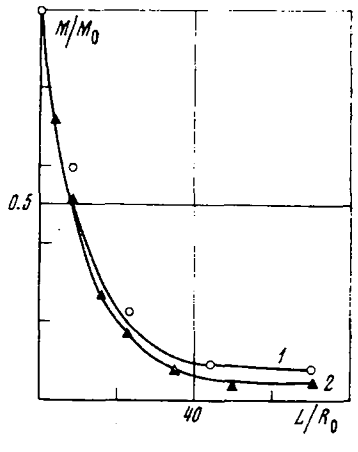

Fig. 4

eled; the circles denote the experimental points. The curve shows that the vortex does contain the two regions mentioned above. The solid line 1 has been drawn in accordance with Eq. (1.5), which in this case takes the form

$$M / M_0 = 0.077 + 0.923 (1 + \alpha L / R_0)^{-14.2}$$

The numbers 0.077 and 0.923 express respectively the ratios of the initial impurity mass at the core and in the atmosphere to the initial mass in the whole vortex; the other numerical values in this series were R_0 = 1.36 cm, $\alpha = 7 \cdot 10^{-3}$, u_0 = 1.5 m/sec, d = 2 cm, l = 6.6 cm, while $\beta = 0.1$.

The photographs enabled us to compare the relative trace sizes δ/R for these vortices and other turbulent vortices; we found that the value, and hence β, was constant. The experiments indicated that $\beta = 0.1$.

The vortices were almost laminar in the initial part of the motion for $Re_0 < 4 \cdot 10^4$; the δ/R for such a vortex in this phase was small relative to that for a turbulent vortex. The vortices became turbulent as they moved further, and a trace of ordinary width was produced behind them. Such a vortex can lose part of its dye during formation. Figure 5 illustrates this. The δ/R and β for such vortices are not constant during the motion.

This laminar feature of a vortex during the initial motion for such Re_0 is controlled by the initial conditions, in particular, l/d; for instance, for $l/d = 2$, we obtained vortices with this property. The other parameters were as above for $l/d = 3.3$. A good agreement with experiment was obtained by assuming that β was constant at 0.068.

It was also possible for vortices to be laminar and lose no dye in the initial part for $Re_0 \sim 10^3$; then turbulence set in, and the subsequent transport was in accordance with Eq. (1.5), in which case the initial parameters should be taken at the moment when turbulence starts.

A deliberate roughening of the nozzle wall caused the jet to be turbulent as it emerged; the resulting vortex was turbulent, and the transport was described by Eq. (1.5) with $\beta = 0.1$; the loss in that case was not dependent on l/d.

There was no appreciable effect on the transport from slight differences in density $[(\rho - \rho')/\rho = 10\%]$ between a source liquid and the medium.

4. The films showed that the relative initial radius of the core a_0/R_0 varied with Re_0 from 1/6 to 1/12, with v_0/V = 1/8-1/20, where v_0 is the initial volume of the core. The form of the relationship is as follows. The mean thickness of the boundary layer is $\Delta \sim l/\sqrt{Re^\circ}$, so $v_0 \sim l^2 d/\sqrt{Re^\circ}$ (Re° is the Reynolds number for the jet from the source). We found that Re° was proportional to Re_0 for $l/d \leq 4$, so for a constant l

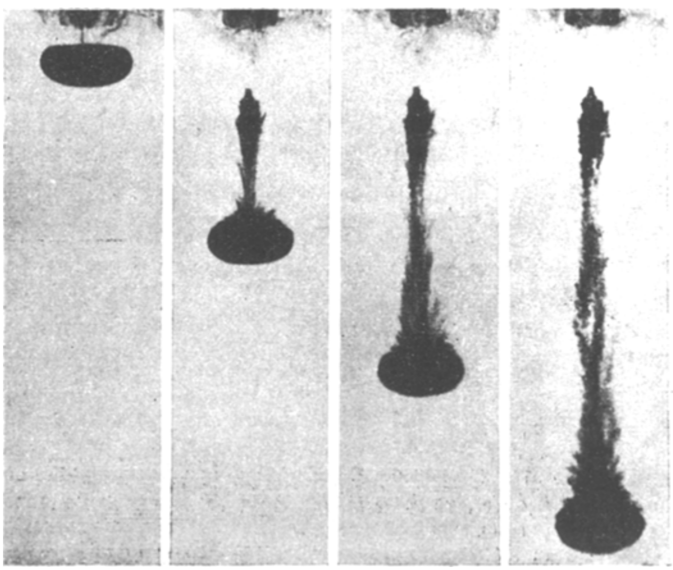

Fig. 5

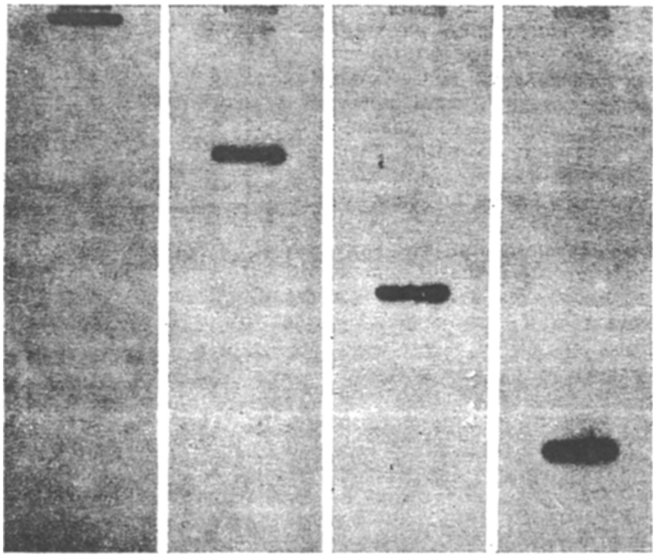

Fig. 6

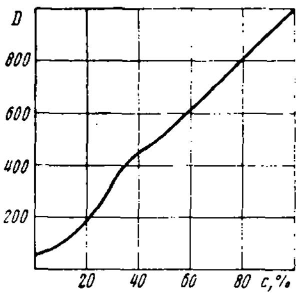

Fig. 7

and d we get for vortices with different Re_0 that

$$v_0\sqrt{Re_0} = \text{const} \tag{4.1}$$

Experimental results confirm this formula.

If a vortex is to transport without loss, it has to fill a volume v_0; for this purpose the impurity should be introduced into the leading part of the bounding layer formed on ejecting the jet. If these conditions are met, one gets results illustrated by Fig. 6.

5. It is found that there is a critical Reynolds number R_* for the given density and particle size at which the transport ceases to be described by Eq. (1.5); if $Re_0 > Re_*$, the material in the core and nearby ceases to be passive, and the centrifugal forces cause the material to be ejected from the core into the atmosphere, the ejection of the particles beginning as the jet leaves the source. For instance, particles of tobacco smoke of size $\sim 10^{-5}$ cm give $Re_* = 8 \cdot 10^3$ for air jets, and so the particle is then not in the region that can carry an impurity without loss and Eq. (1.5) must be replaced by

$$M / M_0 = (1 + \alpha L / R_0)^{-3/\alpha}, \qquad \beta = 0.1 \tag{5.1}$$

6. This method of measurement can provide complete data on the transport, but the method is very laborious. It is simpler and more convenient to photometer the image on the photographic material. One needs a linear relationship between the reading on the linear microphotometer scale D and the variable dye concentration c in order to photometer the entire image at once. One can produce this linearity because the D = D(k) and k = k(c) curves (where k is the absorption coefficient) have curvatures of opposite sign.

One can select the photographic material, the processing conditions, the dye, the initial concentration, the filters, and the recording conditions (stop and delay) to obtain the relationship between D and c shown in Fig. 7. Uniform illumination is also important, and we used daylight lamps placed in contact and illuminating matt glass. This provided a uniform bright background. A Konvas camera was used, the synchronous motor providing a constant recording frequency of 25 frames per second. To prevent blurring, the standard Konvas shutter was replaced by one providing an exposure of 1/250 sec. Blue ink was used to color the solution. We used MZ-2 film and OS-12 filter. We photometered vortices also measured with the trap.

Curves 1 and 2 of Fig. 4 represent the transport recorded with the trap and with the photometer; it is clear that the difference between the two methods is not large, although there is a discrepancy for $L/R_0 > 15$; at that instant the dye concentration in the atmosphere has become small, and the graph of Fig. 7 is not linear for such c. It is also clear that the photometry reduces by a factor of 1.5 the size of the region that transports without loss. Nevertheless, the agreement between the two methods is good, and in similar studies one can use photometry for the purpose provided the above conditions are met.

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